Composition of Dynamical Systems for Estimation of Human Body Dynamics

Sumitra Ganesh[1], Aaron D. Ames[2], and Ruzena Bajcsy[1]

[1] Department of Electrical Engineering and Computer Sciences
University of California, Berkeley, CA 94720
{sumitra,bajcsy}@eecs.berkeley.edu
[2] Control and Dynamical Systems
California Institute of Technology, Pasadena, CA 91125
ames@cds.caltech.edu

Abstract. This paper addresses the problem of estimating human body dynamics from 3-D visual data. That is, our goal is to estimate the state of the system, joint angle trajectories and velocities, and the control required to produce the observed motion from indirect noisy measurements of the joint angles. For a two-link chain in the human body, we show how two independent spherical pendulums can be *composed* to create a behaviorally equivalent double spherical pendulum. Therefore, the estimation problem can be solved in parallel for the low-dimensional spherical pendulum systems and the composition result can be used to arrive at estimates for the higher dimensional double spherical pendulum system. We demonstrate our methods on motion capture data of human arm motion.

1 Introduction and Related Work

The analysis of human motion is motivated by applications such as classification of motion, analysis of motion in activities such as sports and dance, animation and biologically inspired robotic design. Our goal is to extract, from noisy visual observations, a physically meaningful mid-level representation of motion—namely, the joint angle trajectories, angular velocities and joint torques for an assumed nonlinear model. Higher level descriptions, such as motion categories, can be constructed over this representation by using discrete state variables to represent the specific categories. A similar approach was taken in [1] and [2], where switching linear dynamical systems were used for action recognition. Since nonlinear state estimations methods usually scale poorly as the state dimension increases, we propose a more scalable approach to solving the estimation problem. Our approach is based on *composing* estimates for a set of low dimension systems to create a behaviorally equivalent higher dimension system of interest. The paper is organized as follows: Sec. 2 presents our approach and in Sec. 3 we present our results on human arm motion.

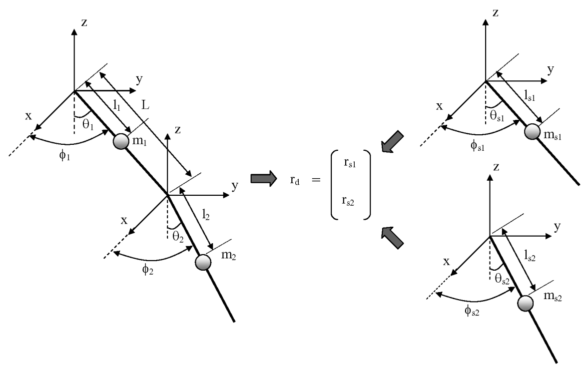

Fig. 1. Equivalent models for the human arm

2 Composition-based Estimation

The human body can be modeled as a set of rigid links connected by joints. The motion of any open chain of links in the human body is described by a set of nonlinear differential equations [3] of the form

$$\underbrace{M(\boldsymbol{r},\boldsymbol{\lambda})}_{\text{mass matrix}}\ \ddot{\boldsymbol{r}} + \underbrace{C(\boldsymbol{r},\dot{\boldsymbol{r}},\boldsymbol{\lambda})}_{\text{coriolis matrix}}\ \dot{\boldsymbol{r}} + \underbrace{N(\boldsymbol{r},\boldsymbol{\lambda})}_{\text{gravity}} = \underbrace{\boldsymbol{\tau}(t)}_{\text{torques}}\ , \tag{1}$$

where $\boldsymbol{r}(t)$ is the vector of joint angles and $\boldsymbol{\lambda}$ denotes the model parameters. We assume that entire mass of a limb is concentrated at its center of mass and model any open chain of links as a series of connected spherical pendulums. The model parameters are designed using anthorpometric data available in [4]. Given observations of the form $\boldsymbol{y}(t) = \boldsymbol{r}(t) + \boldsymbol{\eta}(t)$, where the statistics of the additive noise $\boldsymbol{\eta}(t)$ are known, the goal is to estimate the joint angle trajectories $\boldsymbol{r}(t)$, angular velocities $\dot{\boldsymbol{r}}(t)$ and the required torques $\boldsymbol{\tau}(t)$.

Different dynamical models could produce a given set of joint angle tracjectories $\boldsymbol{r}(t)$. These dynamical models are equivalent with respect to the observations. We refer to the joint angle trajectories as the behavior of the system and use the notation $\mathcal{B}(\Psi|\,\boldsymbol{\lambda},\,\boldsymbol{\tau}\,) \triangleq \boldsymbol{r}$ to denote that the behavior of the system $\Psi = (M, C, N)$ (see (1)), with parameters $\boldsymbol{\lambda}$ and torques $\boldsymbol{\tau}(t)$ is $\boldsymbol{r}(t)$. For instance, consider the joint angle trajectories of a two-link chain such as the human arm—this data could be produced by two independent spherical pendulums actuated by appropriate torques or, equivalently, by an actuated double spherical pendulum.

We now present our composition result for the 2-link case.

Theorem 1. *Let Ψ_{s1} and Ψ_{s2} denote two spherical pendulum models and let Ψ_{d} denote a double spherical pendulum model. Then*

$$\mathcal{B}(\Psi_{\mathrm{d}}|\,\boldsymbol{\lambda}_{\mathrm{d}},\,\boldsymbol{\tau}_{\mathrm{d}}\,) \triangleq \boldsymbol{r}_{\mathrm{d}} = \begin{bmatrix}\boldsymbol{r}_{\mathrm{s1}}\\ \boldsymbol{r}_{\mathrm{s2}}\end{bmatrix} \triangleq \begin{bmatrix}\mathcal{B}(\Psi_{\mathrm{s1}}|\,\boldsymbol{\lambda}_{\mathrm{s1}},\,\boldsymbol{\tau}_{\mathrm{s1}}\,)\\ \mathcal{B}(\Psi_{\mathrm{s2}}|\,\boldsymbol{\lambda}_{\mathrm{s2}},\,\boldsymbol{\tau}_{\mathrm{s2}}\,)\end{bmatrix} \tag{2}$$

$$C_{12} = \mu \begin{bmatrix} (c\theta_2\, s\theta_1 - c\phi\, c\theta_1\, s\theta_2)\dot{\theta}_2 & s\phi\, c\theta_1\, c\theta_2\, \dot{\theta}_2 - c\phi\, c\theta_1\, s\theta_2\, \dot{\phi}_2 \\ + s\phi\, c\theta_1\, c\theta_2\, \dot{\phi}_2 & \\ s\phi\, s\theta_1\, s\theta_2\, \dot{\theta}_2 + c\phi\, c\theta_2\, s\theta_1\, \dot{\phi}_2 & c\phi\, c\theta_2\, s\theta_1\, \dot{\theta}_2 + \mu\, s\phi\, s\theta_1\, s\theta_2\, \dot{\phi}_2 \end{bmatrix}$$

$$C_{21} = \mu \begin{bmatrix} (c\theta_1\, s\theta_2 - c\phi\, c\theta_2\, s\theta_1)\dot{\theta}_1 & - s\phi\, c\theta_1\, c\theta_2\, \dot{\theta}_1 - c\phi\, c\theta_2\, s\theta_1\, \dot{\phi}_1 \\ - s\phi\, c\theta_1\, c\theta_2\, \dot{\phi}_1 & \\ - s\phi\, s\theta_1\, s\theta_2\, \dot{\theta}_1 + c\phi\, c\theta_1\, s\theta_2\, \dot{\phi}_1 & c\phi\, c\theta_1\, s\theta_2\, \dot{\theta}_1 - s\phi\, s\theta_1\, s\theta_2\, \dot{\phi}_1 \end{bmatrix}$$

$$M_{12} = \mu \begin{bmatrix} c\phi\, c\theta_1\, c\theta_2 + s\theta_1\, s\theta_2 & s\phi\, c\theta_1\, s\theta_2 \\ - s\phi\, c\theta_2\, s\theta_1 & c\phi\, s\theta_2\, s\theta_1 \end{bmatrix}$$

Table 1. The matrices M_{12}, C_{12} and C_{21}, with $\mu = m_2 l_2 L$, $s\phi = \sin(\phi_1 - \phi_2)$, $c\theta_1 = \cos\theta_1$, *et cetera.*

for $\boldsymbol{\lambda}_\mathrm{d}$, $\boldsymbol{\lambda}_\mathrm{s1}$ *and* $\boldsymbol{\lambda}_\mathrm{s2}$ *satisfying:*

$$\begin{gathered} \boldsymbol{\lambda}_\mathrm{d} = (m_1, l_1, L, m_2, l_2), \\ \boldsymbol{\lambda}_\mathrm{s1} = \left(\tfrac{(m_1 l_1 + m_2 L)^2}{m_1 l_1^2 + m_2 L^2}, \tfrac{m_1 l_1^2 + m_2 L^2}{m_1 l_1 + m_2 L} \right), \quad \boldsymbol{\lambda}_\mathrm{s2} = (m_2, l_2)\,, \end{gathered} \tag{3}$$

and for $\boldsymbol{\tau}_\mathrm{d}$, $\boldsymbol{\tau}_\mathrm{s1}$ *and* $\boldsymbol{\tau}_\mathrm{s2}$ *satisfying:*

$$\boldsymbol{\tau}_\mathrm{d} = \begin{bmatrix} \boldsymbol{\tau}_\mathrm{s1} + M_{12}(\boldsymbol{r}_\mathrm{s1}, \boldsymbol{r}_\mathrm{s2})\, \ddot{\boldsymbol{r}}_\mathrm{s2} + C_{12}(\boldsymbol{r}_\mathrm{s1}, \boldsymbol{r}_\mathrm{s2}, \dot{\boldsymbol{r}}_\mathrm{s2})\, \dot{\boldsymbol{r}}_\mathrm{s2} \\ \boldsymbol{\tau}_\mathrm{s2} + M_{12}^T(\boldsymbol{r}_\mathrm{s1}, \boldsymbol{r}_\mathrm{s2})\, \ddot{\boldsymbol{r}}_\mathrm{s1} + C_{21}(\boldsymbol{r}_\mathrm{s2}, \boldsymbol{r}_\mathrm{s1}, \dot{\boldsymbol{r}}_\mathrm{s1})\, \dot{\boldsymbol{r}}_\mathrm{s1} \end{bmatrix}, \tag{4}$$

with M_{12}, C_{12} *and* C_{21} *as given in Table 1.*

As a result of this theorem, spherical pendulum models with parameters $\boldsymbol{\lambda}_\mathrm{s1}$ and $\boldsymbol{\lambda}_\mathrm{s2}$ can be used to obtain estimates of upper and lower angle trajectories, velocities and the torques $\boldsymbol{\tau}_\mathrm{s1}(t)$ and $\boldsymbol{\tau}_\mathrm{s2}(t)$, respectively. The *composition* relation (4) can then be used to arrive at an estimate of the torques $\boldsymbol{\tau}_\mathrm{d}(t)$ required for an equivalent double spherical pendulum model. The estimation of the independent spherical pendulum models is done using an auxiliary particle filter [5]. The control utilized in the particle filter is designed to mimic a controller that feedback-linearizes and uses Linear-Quadratic optimal control [6] to track the observations. This structure also provides us with angular acceleration estimates which are used in the composition.

3 Results

We tested our approach on motion capture data from the Carnegie Mellon motion capture database. The motion capture data consisted of human arm motion sampled at 120 Hz. The joint angles extracted from the motion capture data were used as the ground truth reference. Gaussian noise was added to the data to simulate noisy observations. The standard deviation of the observation noise was fixed at 0.2σ, where σ is the standard deviation of the reference. An auxiliary particle filter with 1000 particles was used for the estimation. The results for one action are presented in Figs. 2-4.

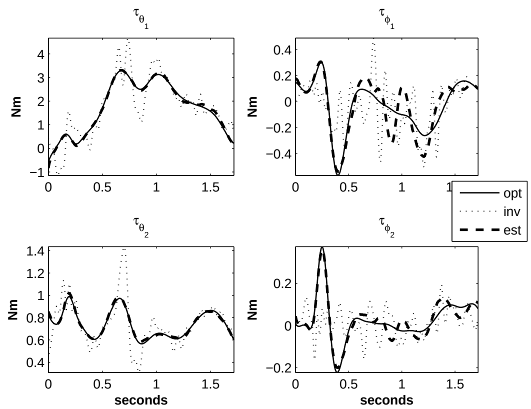

Fig. 2. Double Spherical Pendulum Torques $\boldsymbol{\tau}_{\mathrm{d}}$ for Arm. *est*-torques estimated by composition, *opt*-torques applied by an optimal controller that tracks the reference, *inv*-torques obtained by differentiating clean reference and plugging into equations of motion

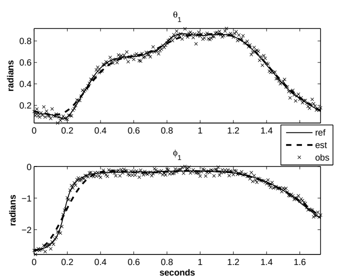

Fig. 3. Estimation of Upper Arm Angles

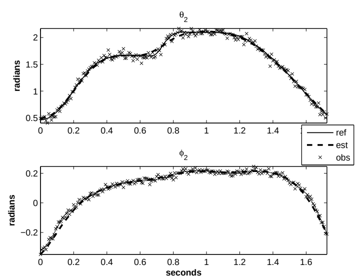

Fig. 4. Estimation of Lower Arm Angles

References

1. Bregler, C., Malik, J.: Learning and recognizing human dynamics in video sequences. In: IEEE Conference on Computer Vision and Pattern Recognition. (1997) pp 568–674
2. Del Vecchio, D., Murray, R., Perona, P.: Decomposition of human motion into dynamics based primitives with application to drawing tasks. Automatica **39(12)** (2003) 2085–2098
3. Murray, R., Sastry, S., Li, Z.: A Mathematical Introduction to Robotic Manipulation. CRC Press (1994)
4. Winter, D.A.: Biomechanics and Motor Control of Human Movement. Wiley (1990)
5. Pitt, M.K., Shephard, N.: Filtering via simulation: Auxiliary particle filters. Journal of the American Statistical Association **94**(446) (1999) 590–9
6. Lewis, F., Syrmos, V.: Optimal Control. Wiley-Interscience (1995)