Human-Inspired Control of Bipedal Robots via Control Lyapunov Functions and Quadratic Programs

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ABSTRACT
This paper briefly presents the process of formally achieving bipedal robotic walking through controller synthesis inspired by human locomotion. Motivated by the hierarchical control present in humans, we begin by viewing the human as a “black box” and describe outputs, or virtual constraints, that appear to characterize human walking. By considering the equivalent outputs for the bipedal robot, a nonlinear controller can be constructed that drives the outputs of the robot to the outputs of the human; moreover, the parameters of this controller can be optimized so that stable robotic walking is provably achieved while simultaneously producing outputs of the robot that are as close as possible to those of a human. Finally, considering a control Lyapunov function based representation of these outputs allows for the class of controllers that provably achieve stable robotic walking can be greatly enlarged. The end result is the generation of bipedal robotic walking that is remarkably human-like and is experimentally realizable, as evidenced by the implementation of the resulting controllers on multiple robotic platforms.

Categories and Subject Descriptors
J.2 [Physical Sciences and Engineering]: [engineering, mathematics and statistics]; G.1.6 [Numerical Analysis]: Optimization—constrained optimization

Keywords
hybrid systems, bipedal robotic walking, nonlinear dynamics and control, control Lyapunov functions, optimization

1. HUMAN-INSPIRED CONTROL
Humans have the ability to walk with deceptive ease, navigating everything from daily environments to uneven and uncertain terrain with efficiency and robustness. Despite the similarity with which humans appear to locomote, locomotion is inherently complex due to highly nonlinear dynamics and forcing. Yet there is evidence to suggest that humans utilize a hierarchical subdivision among cortical control, central pattern generators in the spinal column, and proprioceptive sensory feedback. This indicates that when humans perform motion primitives, potentially simple and characterizable control strategies are implemented.

With the goal of representing human walking in a form amenable to control synthesis, consider a human output combination: \( Y^H = (Q, y_1^H, y_2^H) \), consisting of the configuration space of a robot, \( Q \subset \mathbb{R}^n \), a velocity modulating output \( y_1^H : Q \to \mathbb{R}^n \), position modulating outputs \( y_2^H : Q \to \mathbb{R}^{n-1} \) given by \( y_2^H(\theta) = [y_2^H(\theta)]_{i \in O} \) with \( O \) an indexing set for \( y_2^H \). By considering human walking data, specific human output combinations appear to kinematically characterize human walking through the canonical walking function (CWF) [2, 3, 4]:
\[
y_{H}(t, a) = e^{-a t}(\alpha_1 \cos(\alpha_2 t) + \alpha_3 \sin(\alpha_2 t)) + \alpha_5. \tag{1}
\]
In particular, consider human data taken over one step of a walking gait at discrete times \( t[k], k \in \{1, 2, \ldots, N\} \), yielding discrete angle measurements of the human \( \theta^H[k] \in Q \) as appropriately mapped to the robot model. For the proper human output combination and choice of parameters, \( \alpha = (\nu, (\alpha_i)_{i \in O}) \), we find that (see Fig. 1(b)): \( y_2^H(\theta^H[k]) \approx v^H[k] \), and \( y_2^H(\theta^H[k]) \approx [y_H(t^H[k], \alpha_i)]_{i \in O} \). This implies that a proper human output combination characterizes the behavior of human walking at a kinematic level to be that of a linear mass-spring-damper system.

Human output combinations can be used to synthesize nonlinear controllers by considering the following human-inspired outputs:
\[
y_1(\theta, \dot{\theta}) = dy_1^H(\theta)\dot{\theta} - v, \tag{2}
y_2(\theta) = y_2^H(\theta) - [y_H(\tau(\theta), \alpha_i)]_{i \in O}. \tag{3}
\]
where \( \tau(\theta) \) is a parameterization of time based upon the velocity modulating output \( y_1^H \). Choosing a classic input/output (IO) linearizing controller of the form [2, 3]:
\[
u = A^{-1}(-L_y1_{L_y^2} + \mu), \quad \mu = \begin{bmatrix} -2k y_1 & -2k y_2 & -2k y_2 \end{bmatrix}^T, \tag{4}
\]
where \( A \) is the decoupling matrix and the dependency on \( \theta \) and \( \dot{\theta} \) has been suppressed. This controller drives the outputs to zero exponentially at a user-defined rate of \( \varepsilon \), i.e., \( y_1 \to 0 \) and \( y_2 \to 0 \). Intuitively, driving these outputs to zero drives the outputs of the robot—both velocity and position modulating—to the outputs of a human as represented by a constant for the velocity modulating output, and the CWF for the position-based outputs.

The parameters, \( \alpha \), of the CWF that best fit the human data will not generally result in robotic walking due to differences between the robot and human. Therefore, the final step in human-inspired control synthesis is generating parameters that simultaneously generate provably stable robotic walking while producing outputs that are as
2. CONTROL LYAPUNOV FUNCTIONS

The IO formulation of human-inspired control yields provably stable walking, but does so through a specific choice of the dynamics for the IO linearized system as given by $\mu$ in (4). This can be disruptive to the natural dynamics of the robot, potentially resulting in inefficient walking. This motivates the consideration of a broader class of controllers that yield provably stable behavior—those obtained from rapidly exponentially stabilizing control Lyapunov functions (RES-CLF) [5], defined as functions $V_i : X \rightarrow \mathbb{R}$, dependent on a parameter $\varepsilon > 0$ and satisfying, for $c_1, c_2, c_3 > 0$:

$$c_1\|x\|^2 \leq V_i(x) \leq \varepsilon^2 c_2\|x\|^2$$ \hspace{1cm} (5)

$$\inf_{u \in U} [L_f V_i(x, z) + L_y V_i(x, z) u + \varepsilon c_3 V_i(x)] \leq 0$$ \hspace{1cm} (6)

where here $x \in X$ consists of the controlled (or output) states and $z \in \mathcal{Z}$ are the uncontrolled states, e.g., $x = (y_1, y_2)$ for AMBER 1 and $x = (\hat{y}_1, y_2, \hat{y}_2)$ for AMBER 2.

A RES-CLF naturally yields a class of controllers:

$$K_i(x, z) = \{ u \in U : L_f V_i(x, z) + L_y V_i(x, z) u + \varepsilon c_3 V_i(x) \},$$

for which choosing any Lipschitz continuous controller $u \in K_i$ implies a stable walking gait, i.e., an exponentially stable hybrid periodic orbit, for $\varepsilon > 0$ sufficiently large and assuming (partial) hybrid zero dynamics $\Delta(S \cap \mathcal{P}_{\alpha, *}) \subset \mathcal{P}_{\alpha, *}$ [5]. The control law (4) provides a specific example of such a controller, so the RES-CLF formulation greatly increases the class of controllers that yield robotic walking. Of particular interest is the min-norm controller:

$$m(x, z) = \arg\min\{\|u\| : u \in K_i(x, z)\},$$

which generates the minimal control effort needed to achieve rapid exponential convergence. This can be naturally expressed as a quadratic program in $u$, allowing for additional control constraints to be added such as torque bounds, and therefore promises to further bridge the gap between theory and experimental realization.

3. REFERENCES


