Human-Inspired Control of Bipedal Robots via Control Lyapunov Functions and Quadratic Programs

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ABSTRACT

This paper briefly presents the process of formally achieving bipedal robotic walking through controller synthesis inspired by human locomotion. Motivated by the hierarchical control present in humans, we begin by viewing the human as a "black box" and describe outputs, or virtual constraints, that appear to characterize human walking. By considering the equivalent outputs for the bipedal robot, a nonlinear controller can be constructed that drives the outputs of the robot to the outputs of the human; moreover, the parameters of this controller can be optimized so that stable robotic walking is provably achieved while simultaneously producing outputs of the robot that are as close as possible to those of a human. Finally, considering a control Lyapunov function based representation of these outputs allows for the class of controllers that provably achieve stable robotic walking can be greatly enlarged. The end result is the generation of bipedal robotic walking that is remarkably human-like and is experimentally realizable, as evidenced by the implementation of the resulting controllers on multiple robotic platforms.

Categories and Subject Descriptors

J.2 [**Physical Sciences and Engineering**]: [engineering, mathematics and statistics]; G.1.6 [**Numerical Analysis**]: Optimization—*constrained optimization*

Keywords

hybrid systems, bipedal robotic walking, nonlinear dynamics and control, control Lyapunov functions, optimization

1. HUMAN-INSPIRED CONTROL

Humans have the ability to walk with deceptive ease, navigating everything from daily environments to uneven and uncertain terrain with efficiency and robustness. Despite the simplicity with which humans appear to ambulate, locomotion is inherently complex due to highly nonlinear dynamics and forcing. Yet there is evidence to suggest that humans utilize a hierarchical subdivision among cortical control, central pattern generators in the spinal column, and proprioceptive sensory feedback. This indicates that when humans perform motion primitives, potentially simple and characterizable control strategies are implemented.

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With the goal of representing human walking in a form amenable to control synthesis, consider a *human output combination*: $Y^H = (Q, y_1^H, y_2^H)$, consisting of the configuration space of a robot, $Q \subset \mathbb{R}^n$, a velocity modulating output $y_1^H : Q \to \mathbb{R}$, position modulating outputs $y_2^H : Q \to \mathbb{R}^{n-1}$ given by $y_2^H(\theta) = [y_2^H(\theta)_i]_{i \in O}$ with O an indexing set for y_2^H. By considering human walking data, specific human output combinations appear to kinematically characterize human walking through the *canonical walking function (CWF)* [2, 3, 4]:

$$y_H(t, \alpha) = e^{-\alpha_4 t}(\alpha_1 \cos(\alpha_2 t) + \alpha_3 \sin(\alpha_2 t)) + \alpha_5. \quad (1)$$

In particular, consider human data taken over one step of a walking gait at discrete times $t[k]$, $k \in \{1, \ldots, N\}$, yielding discrete angle measurements of the human $\theta^H[k] \in Q$ as appropriately mapped to the robot model. For the proper human output combination and choice of parameters, $\alpha = (v, (\alpha_i)_{i \in O})$, we find that (see Fig. 1(b)): $y_1^H(\theta^H[k]) \approx vt^H[k]$, and $y_2^H(\theta^H[k]) \approx [y_H(t^H[k], \alpha_i)]_{i \in O}$. This implies that a proper human output combination characterizes the behavior of human walking at a kinematic level to be that of a linear mass-spring-damper system.

Human output combinations can be used to synthesize nonlinear controllers by considering the following *human-inspired outputs*:

$$\begin{aligned} y_1(\theta, \dot{\theta}) &= dy_1^H(\theta)\dot{\theta} - v, & (2)\\ y_2(\theta) &= y_2^H(\theta) - [y_H(\tau(\theta), \alpha_i)]_{i \in O}. & (3) \end{aligned}$$

where $\tau(\theta)$ is a parameterization of time based upon the velocity modulating output y_1^H. Choosing a classic input/output (IO) linearizing controller of the form [2, 3]:

$$u = A^{-1}\left(-\begin{bmatrix} L_f y_1 \\ L_f^2 y_2 \end{bmatrix} + \mu\right), \quad \mu = \begin{bmatrix} -\varepsilon y_1 \\ -2\varepsilon \dot{y}_2 - \varepsilon^2 y_2 \end{bmatrix}, \quad (4)$$

where A is the decoupling matrix and the dependency on θ and $\dot{\theta}$ has been suppressed. This controller drives the outputs to zero exponentially at a user-defined rate of ε, i.e., $y_1 \to 0$ and $y_2 \to 0$. Intuitively, driving these outputs to zero drives the outputs of the robot—both velocity and position modulating—to the outputs of a human as represented by a constant for the velocity modulating output, and the CWF for the position-based outputs.

The parameters, α, of the CWF that best fit the human data will not generally result in robotic walking due to differences between the robot and human. Therefore, the final step in human-inspired control synthesis is generating parameters that simultaneously generate provably stable robotic walking while producing outputs that are as

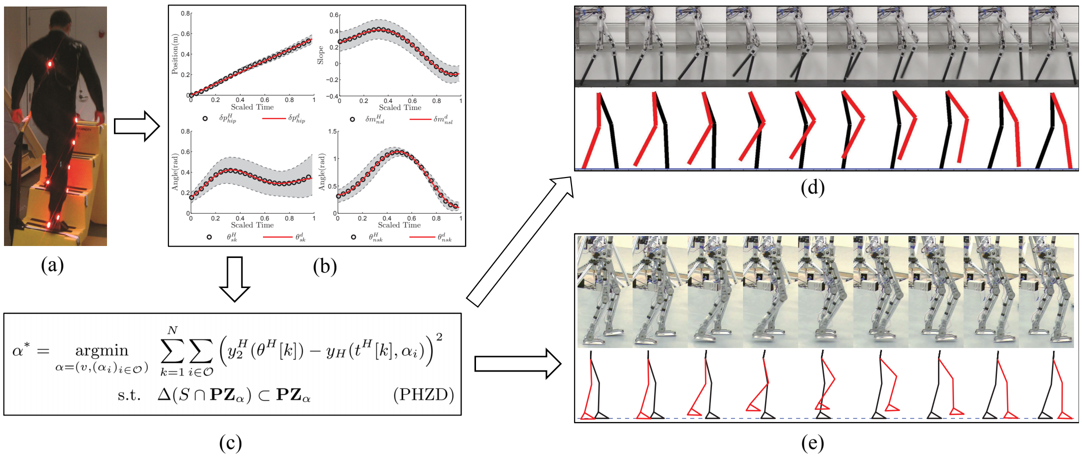

Figure 1: The process of generating robotic walking through human-inspired control. Human output data (b) is calculated from human walking data (a) and utilized in the human-inspired optimization (c). The end result is provably stable robotic walking which has been experimentally realized on AMBER 1 (d) and AMBER 2 (e). Videos of the walking can be seen at [1]

close as possible to those of the human. This is achieved by solving the human-inspired optimization problem shown in Fig. 1(c), where $\mathbf{PZ}_\alpha$ is the partial zero dynamics surface for which the relative degree 2 outputs $y_2 = 0$ for all time, S is the switching surface (or the set of the points where the non-stance foot impacts the ground) and Δ is the impact equation that determines the change in velocity of the system when the foot strikes the ground. Importantly, the solution to this optimization implies that the system will evolve on the PHZD surface defined by $\mathbf{PZ}_{\alpha^*}$, reducing the evolution of the robot to a two-dimensional space. This low-dimensional representation allows for the formal construction of a stable robotic walking gait satisfying physical constraints (such as torque and friction constraints) that can be computed in closed form. The end result is the fast and efficient synthesis of walking gaits that are experimentally realizable. These methods have been utilized to experimentally achieve robotic walking on three robotic platforms, including the 2D bipedal robots AMBER 1.0 and AMBER 2.0 and the 3D humanoid robot NAO [1]. Examples of the walking achieved are shown in Figure 1.

2. CONTROL LYAPUNOV FUNCTIONS

The IO formulation of human-inspired control yields provably stable walking, but does so through a specific choice of the dynamics for the IO linearized system as given by μ in (4). This can be disruptive to the natural dynamics of the robot, potentially resulting in inefficient walking. This motives the consideration of a broader class of controllers that yield provably stable behavior—those obtained from *rapidly exponentially stabilizing control Lyapunov functions (RES-CLF)* [5], defined as functions $V_\varepsilon : X \to \mathbb{R}$, dependent on a parameter $\varepsilon > 0$ and satisfying, for $c_1, c_2, c_3 > 0$:

$$c_1\|x\|^2 \le V_\varepsilon(x) \le \varepsilon^2 c_2 \|x\|^2 \qquad (5)$$

$$\inf_{u \in U} \left[L_f V_\varepsilon(x,z) + L_g V_\varepsilon(x,z)u + \varepsilon c_3 V_\varepsilon(x)\right] \le 0 \qquad (6)$$

where here $x \in X$ consists of the controlled (or output) states and $z \in Z$ are the uncontrolled states, e.g., $x = (y_2, \dot{y}_2)$ for AMBER 1 and $x = (\dot{y}_1, y_2, \dot{y}_2)$ for AMBER 2.

A RES-CLF naturally yields a class of controllers:

$$K_\varepsilon(x,z) = \{u \in U : L_f V_\varepsilon(x,z) + L_g V_\varepsilon(x,z)u + \varepsilon c_3 V_\varepsilon(x)\},$$

for which choosing any Lipschitz continuous controller $u_\varepsilon \in K_\varepsilon$ implies a stable walking gait, i.e, an exponentially stable hybrid periodic orbit, for $\varepsilon > 0$ sufficiently large and assuming (partial) hybrid zero dynamics $\Delta(S \cap \mathbf{PZ}_{\alpha^*}) \subset \mathbf{PZ}_{\alpha^*}$ [5]. The control law (4) provides a specific example of such a controller, so the RES-CLF formulation greatly increases the class of controllers that yield robotic walking. Of particular interest is the min-norm controller:

$$m(x,z) = \operatorname{argmin}\{\|u\| : u \in K_\varepsilon(x,z)\},$$

which generates the minimal control effort needed to achieve rapid exponential convergence. This can be naturally expressed as a quadratic program in u, allowing for additional control constraints to be added such as torque bounds, and therefore promises to further bridge the gap between theory and experimental realization.

3. REFERENCES

[1] YouTube page for AMBER lab. `http://www.youtube.com/ProfAmes`.

[2] A. D. Ames. First steps toward automatically generating bipedal robotic walking from human data. In *Robotic Motion and Control 2011*, volume 422 of *LNICS*, pages 89–116. Springer, 2012.

[3] A. D. Ames. First steps toward underactuated human-inspired bipedal robotic walking. In *IEEE International Conference on Robotics and Automation*, St. Paul, MN, 2012.

[4] A. D. Ames, E. A. Cousineau, and M. J. Powell. Dynamically stable robotic walking with NAO via human-inspired hybrid zero dynamics. In *Hybrid Systems: Computation and Control*, Beijing, 2012.

[5] A. D. Ames, K. Galloway, and J. W. Grizzle. Control Lyapunov Functions and Hybrid Zero Dynamics. In *51st IEEE Conference on Decision and Control*, Maui, HI, 2012.