A Hybrid Systems and Optimization-Based Control Approach to Realizing Multi-Contact Locomotion on Transfemoral Prostheses

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Abstract—This paper presents a systematic methodology utilizing multi-domain hybrid system models and optimization based controllers to achieve human-like multi-contact prosthetic walking experimentally on a custom-built prosthesis: AMPRO. Inspired by previous work that realized multi-contact locomotion on a bipedal robot AMBER2, a hybrid system based optimization problem is proposed leveraging the framework of multi-domain hybrid systems. Utilizing a reference human gait coupled with physical constraints, the end result of this optimization problem is stable multi-contact prosthetic gaits that can be implemented on the prostheses directly. Leveraging control methods that stabilize bipedal walking robots—control Lyapunov function based quadratic programs coupled with variable impedance control—an online optimization-based controller is formulated to realize the designed gait in both simulation and experimentally on AMPRO. Improved tracking and energy efficiency are seen when this methodology is implemented experimentally. Additionally, the resulting multi-contact prosthetic walking captures the essentials of natural human walking both kinematically and kinetically.

I. INTRODUCTION
Due to the large number of lower-limb amputees needing powered robotic assistive devices [10], there is an increasing demand for more efficient and naturally moving devices. Multi-contact foot behaviors, which are naturally exhibited in human walking [3], [23], are crucial to achieving these goals [14], [17]. This requires the modeling of these behaviors—which is complex due to the changing contact points—and control that guarantees stability and efficiency. As a means to address this need, and the challenges in achieving these behaviors, this paper considers both the modeling of multi-contact locomotion, as it relates to prostheses, and correspondingly constructs controllers for these multi-domain hybrid systems. These contributions are realized experimentally on a novel prosthesis: AMPRO.

Motivated by the advantages and challenges of multi-contact locomotion with foot motion, it has been studied actively in the control and robotic field in the recent decade with the broader goal of achieving close human-like locomotion. In this setting, methods utilizing the popular Zero Moment Point, including gait pattern generation and gait planning methods, are adopted to design the foot trajectory specifically for multi-contact foot behavior in [7], [11], [12]. However, the resulting gait only has foot roll during the double support phase, i.e., the foot remains flat during the stance phase. Simulated robotic walking with significant toe push can be found in [9], [24], in which the authors show that the walking gait with toe push helps reduce torque and achieve faster walking speeds. Different from the above approaches, previous work by the authors [29] started with a hybrid system model motivated by human locomotion, and proposed a novel multi-domain optimization problem which embeds this multi-contact feature into gait design to generate human-like locomotion in a manner that is both formally correct as well as physically realizable. This was combined with a trajectory reconstruction method, with the end result being successful experimental realization of stable human-like multi-contact locomotion on a 7-link 2D bipedal robot AMBER2 as seen in Fig. 1 (see video at [1]).

The main goal of this paper is to extend the framework for achieving multi-contact robotic walking [29], as motivated by previous work by the authors in translating simpler locomotion behaviors to prosthesis [31], to achieve natural prosthetic walking. More explicitly, through utilizing an opt-
imal prosthetic gait that is generated based on the hybrid system model of multi-domain locomotion, the main contribution of this paper is the development of an optimization-based controller and its realization of multi-contact prosthetic walking on the custom-built prosthesis: AMPRO. This contribution is accomplished through two novel steps.

The first step is to generate an optimal multi-contact prosthetic gait utilizing a multi-domain hybrid system model. The traditional approach, variable impedance control is a common framework for lower-limb prostheses [8], [22]; by dividing a single step into several phases, a multi-contact prosthetic gait (with heel strike and toe push) can be designed explicitly by carefully choosing control parameters for each phase. A shortcoming of this methodology is that clinicians choose these parameters by trial and error for each individual or each gait, which can be costly and time consuming [4]. This work takes a different approach by constructing a multi-domain hybrid system model for a “robot” with anthropomorphic parameters. Utilizing a low-cost motion capture system for healthy reference human locomotion data collection, a multi-domain optimization problem subject to specific constraints is then proposed to generate a customized stable multi-contact prosthetic gait. The end result is an automatically generated prosthetic gait, which is both optimal and directly implementable for the prosthetic device, therefore, essentially eliminating the requirement of parameter tuning.

Utilizing control methods that stabilize bipedal walking robots, in particular control Lyapunov functions [6], the second step is to formulate a quadratic program based controller that achieves rapidly exponential convergence of virtual constraints subject to actuator bounds. When coupled with impedance control as a feed-forward term, the end result is a model independent quadratic programming (MIQP) controller that is able to achieve better tracking and improved energy efficiency on prostheses. The resulting real-time optimization based controller was experimentally realized on the custom-built prosthesis AMPRO with the end result being multi-contact prosthetic locomotion.

II. MULTI-CONTACT PROSTHETIC GAIT GENERATION

This section reviews the multi-contact behavior embedded in human locomotion. A motion capture system with inertial measurement units (IMUs) is used to capture the human locomotion data. With the goal of designing a prosthetic gait utilizing robot models, a multi-domain bipedal hybrid system is constructed for a “robot” with anthropomorphic parameters. Based on this model and the reference human locomotion data obtained with the IMUs, a human-inspired optimization problem is constructed for the hybrid system model of multi-contact locomotion [29]; the end result is a customized prosthetic gait that (a) yields theoretically provable stability, (b) captures the essential multi-contact behaviors of healthy human walking and (c) suits the specific test subject wearing the prosthetic device.

A. Multi-Contact Human Locomotion

We begin our analysis of the walking pattern of a normal leg by breaking a single step cycle into different distinct phases based on the points of the feet that are in contact with the ground. Utilizing the domain breakdown method discussed in [30], three domains (i.e., sub-phases) of a single step are considered as motivated by the multi-contact walking achieved on the bipedal robot AMBER2 [29]. Based on the actuation type and contact points, we denote the three domains as over-actuated domain, oa (with the stance heel and swing toe in contact with the ground), fully-actuated domain, fa (with the stance heel and toe in the ground) and under-actuated domain, ua (with only stance toe in the ground) as shown in Fig. 2. The switching between domains is triggered by the change of contact points on the feet.

With the goal of obtaining a specific reference gait (i.e., the gait from a healthy subject who has similar anthropomorphic parameters as the amputee), a low-cost inertial motion capture system with IMUs is developed to collect the healthy human planar locomotion data. A model based Extended Kalman Filter (EKF) [21] is used to obtain accurate joint angle information about the human subject [31]. During the experiments, the subject was asked to walk along a straight line in a normal self-selected cadence for several steps, the data of which are averaged to yield the unique reference trajectories for the optimization problem that will be discussed later.

B. Multi-Contact Robot Model

Considering the changes of foot contact points over a gait cycle (lifting and striking of the heel and toe), a hybrid system model is developed, i.e., a model with both continuous and discrete dynamics. Formally, a multi-domain walking gait can be modeled as a hybrid control system [25] given by the following tuple:

\[
\mathcal{H} \in \{\Gamma, D, U, S, \Delta, FG\},
\]

where

- \( \Gamma = (V, E) \) is a directed cycle, with vertices \( V = \{oa, fa, ua\} \); and edges \( E = \{e_1 = \{oa \rightarrow fa\}, e_2 = \{fa \rightarrow ua\}, e_3 = \{ua \rightarrow oa\}\} \),
The detailed explanation of each element of this hybrid system is omitted here and can be found in [16], [29].

The configuration space $Q$ of the robot is characterized by the generalized coordinates: $\theta = \{p_x, p_z, \varphi_0, \theta_b\}$, where the extended coordinates $\{p_x, p_z, \varphi_0\}$ represent the position and rotation angle of the body fixed frame $R_b$ with respect to the world frame $R_0$; and $\theta_b = \{\theta_{iu}, \theta_{ik}, \theta_{ih}, \theta_{nsh}, \theta_{nzk}, \theta_{nsl}\}$ denotes the body coordinates shown in Fig. 3. The dynamics on each domain can be obtained from general “unpinned” model through the use of holonomic constraints [18], [26]:

$$
M(\theta) \dot{\theta} + H(\theta, \dot{\theta}) = B(\theta)u + J_v(\theta)F_v(\theta, \dot{\theta}, u),
$$

$$
J_v(\theta)\dot{\theta} + J_i(\theta)\dot{\theta} = 0,
$$

(2)

where $M(\theta) \in \mathbb{R}^{9 \times 9}$ is the inertial matrix, and $H(\theta, \dot{\theta}) \in \mathbb{R}^{9 \times 1}$ contains the terms resulting from the Coriolis effect $C(\theta, \dot{\theta})\dot{\theta}$ and the gravity vector $G(\theta)$. $B(\theta)$ denotes the torque distribution matrix. $J_v(\theta)$ is the Jacobian of specific contact points of the corresponding domain $\nu \in V$. $F_v(\theta, \dot{\theta})$ are the reaction forces due to the holonomic constraints and defined specifically based on the contact conditions of the heel and toe [18]. With the notation $x = (\theta, \dot{\theta})$, the affine control system $\dot{x} = f_v(x) + g_v(x)u$ for each domain $D_v$ with $\nu \in V$ can be obtained by reformulating (2) [20]. The discrete behavior of impacts (including toe strike and heel strike of the multi-contact model) is modeled with the perfectly plastic impact assumption; more details can be found in [13], [29].

C. Human-Inspired Outputs

We begin by viewing the complex human locomotion system as a “black box.” Therefore, the goal of achieving human-like robotic walking becomes to drive the actual robot outputs $y_v(\theta)$ to the desired human outputs $y_v(t, \alpha)$ that are represented by a specific walking function (see [29]). In particular, a total of 7 outputs are of interest for the multi-domain 7-link bipedal robot, which yields the human-inspired outputs [5]:

$$
y(\theta, \dot{\theta}, \alpha) = \begin{bmatrix} y_1(\theta, \dot{\theta}, \alpha) \\ y_2(\theta, \alpha) \end{bmatrix} = \begin{bmatrix} y_1^L(\theta, \dot{\theta}) - v_{hip} \\ y_2^L(\theta) - y_2^L(\rho(\theta), \alpha) \end{bmatrix},
$$

(3)

where $y_1(\theta, \dot{\theta}, \alpha)$ and $y_2(\theta, \alpha)$ are the relative degree one output and relative degree two outputs, respectively. The parameter set $\alpha$ is the grouped parameters of all the outputs consisting of both the relative degree one output and relative degree two outputs for a complete step cycle [29]. Based on the actuation type in each domain $D_v$ with $\nu \in V$, the corresponding components $\alpha_v$ of $\alpha$ will be utilized to define the human-inspired outputs. Importantly, for a specific output, the parameters will be kept unchanged for all the domains.

Upon observation of multi-contact human locomotion data, the linearized forward hip position, $\delta p_{\text{hip}}(\theta)$, was discovered to increase linearly through the progress of a step [15]; this motives the following phase variable:

$$
\rho(\theta) = (\delta p_{\text{hip}}(\theta) - \delta p_{\text{hip}}^0)/v_{\text{hip}},
$$

(4)

aiming to remove the dependency of time [5], [26]; here $\delta p_{\text{hip}}^0(\theta)$ is the hip position at the beginning of a step.

Partial Hybrid Zero Dynamics. The human-inspired controller as discussed in [5] can be utilized to drive both $y_1 \to 0$ and $y_2 \to 0$ in a provably exponentially stable fashion for the continuous dynamics. However, the robot will “bounce-off” the designed trajectory when impacts occur. This motivates the introduction of the partial hybrid zero dynamics (PHZD) constraints aiming to produce a parameter set $\alpha$ that ensures the tracking of relative degree two outputs remain invariant through impacts. In particular, with the partial zero dynamics (PZD) surface defined as:

$$
\text{PZ}_\alpha = \{(\theta, \dot{\theta}) \in Q : y_2(\theta, \alpha) = 0, L_f y_2(\theta, \alpha) = 0\},
$$

(5)

the general PHZD constraints can be stated abstractly as:

$$
\Delta(S \cap \text{PZ}_\alpha) \subseteq \text{PZ}_\alpha,
$$

(PHZD)

which are required to be valid through all three discrete transitions as illustrated in (1). Explicitly, the three sets of PHZD constraints can be stated as:

$$
\Delta_{oa\rightarrow fa}(S_{oa\rightarrow fa} \cap \text{PZ}_\alpha) \subseteq \text{PZ}_\alpha^f,
$$

(PHZD1)

$$
\Delta_{fa\rightarrow ss}(S_{fa\rightarrow ss} \cap \text{PZ}_\alpha^f) \subseteq \text{PZ}_\alpha^s,
$$

(PHZD2)

$$
\Delta_{ss\rightarrow oo}(S_{ss\rightarrow oo} \cap \text{PZ}_\alpha^s) \subseteq \text{PZ}_\alpha^o.
$$

(PHZD3)

The detailed construction of these constraints requires the explicit explanation of techniques such as the reduced order hybrid zero dynamics, inverse kinematics and PHZD reconstructions, which are omitted here for simplicity of the paper structure. The details can be found in [16], [29].

D. Multi-Contact Prosthetic Gait Design

By enforcing the PHZD constraints discussed above, a multi-contact human-inspired optimization is utilized to design walking trajectories that are both provably stable and human-like [5], [29]. More importantly, physical constraints.
III. PROSTHETIC CONTROLLER DESIGN

This section begins by briefly introducing a real-time optimization-based prostheses controller, which has been proposed in [28] and validated in [31] for achieving flat-foot prosthetic walking on a custom built prosthesis, AMRPO. With the multi-contact trajectories in hand, the controller is then implemented to achieve prosthetic walking in simulation at the end of this section.

A. MIQP+Impedance Control

In previous work [31], the authors proposed a novel prosthetic controller that combines the rapidly exponentially stabilizing control Lyapunov functions (RES-CLFs) based quadratic program control [6] with impedance control in an effort to achieve better tracking and improved energy efficiency on prostheses. In particular, using the human-inspired feedback linearization controller [5], equation (2) can be converted to a linear form as follows:

$$\eta \left[ \begin{array}{c} \eta \left( \phi_{p} \right) \\eta \left( \theta_{p} \right) \end{array} \right] + \left[ \begin{array}{c} \frac{1}{2} I \\frac{1}{2} I \end{array} \right] \eta : \left[ \begin{array}{c} \frac{1}{2} I \\frac{1}{2} I \end{array} \right] \eta := \eta^T P e \eta, \quad (8)$$

where $\eta = (y_p; y_p) \in \mathbb{R}^{4\times1}$ with $y_p = (\theta_p; \theta_p)^T$ the angles for the prosthetic ankle joint and knee joint, respectively. Leveraging the Continuous Algebraic Riccati Equations with solution $P = P^T > 0$, allows for the construction of a RES-CLF [6] given as:

$$L_F V_F(\eta) + L_G V_G(\eta) \mu \leq -\frac{\gamma}{\varepsilon} V_F(\eta), \quad (9)$$

where $L_F V_F(\eta)$ and $L_G V_G(\eta)$ are the corresponding Lie derivatives of the Lyapunov function (8) relative to (7). Particularly, an optimal (point-wise) $\mu$ could be found by turning this condition into a quadratic problem (QP) while enforcing a relaxation term $\mu^{imp}$ into the construction for the total hardware torque bounds, which yields the following model independent quadratic program plus impedance control (MIQP+Impedance):

$$\arg\min_{(\delta, \mu^{qp}) \in \mathbb{R}^{4\times1}} p \delta^2 + \mu^{qp^T} \mu^{qp} \quad (10)$$

subject to $L_F V_F(\eta) + \frac{\gamma}{\varepsilon} V_F(\eta) + L_G V_G(\eta) \mu^{qp} \leq \delta, \quad \text{(CLF)}$

$$\mu^{qp} \leq \mu^{qp}_{\text{Max}}, \quad \text{(Max QP Torque)}$$

$$-\mu^{qp} \leq \mu^{ qp}_{\text{Max}}, \quad \text{(Min QP Torque)}$$

$$\mu^{ qp} \leq \mu^{\text{Max}} - \mu^{imp}, \quad \text{(Max Input Torque)}$$

$$-\mu^{ qp} \leq \mu^{\text{Max}} + \mu^{imp}, \quad \text{(Min Input Torque)}$$

This QP problem yields an optimal controller that regulates the output errors in a rapidly exponentially stable fashion. Additionally, the model independent controller gathers information about the system through the addition of the feedforward impedance control to the input torque. By setting the QP torque bounds $\mu^{ qp}_{\text{Max}}$, we can limit the overshoot problems. We also set the total input torque bounds for the QP problem such that the final optimal input torque will satisfy the hardware torque bounds $\mu^{\text{Max}}$, which is critical for practical implementation.
By leveraging a systematic methodology—including hybrid system models and real-time optimization-based controllers—this paper successfully translated the multi-contact behavior that is intrinsic in human locomotion from bipedal walking on AMBER2 to prosthetic walking on the prosthesis AMPRO. The performance of multiple controllers—utilized to track the generated optimal multi-contact gait—are compared with the real-time optimization

joints in the sagittal plane. Two FlexiForce force sensors are mounted at the toe and heel to detect foot contact, which will be used for stance-swing switching. More details about the design specifications can be found in [31]. To provide a point of human-robotic interaction, two IMUs are mounted on the shin and thigh of the human leg for measuring relative orientation and velocity for both the knee and ankle. In particular, while the human leg is in stance, IMU readings are utilized to compute the forward hip position and forward hip velocity; therefore, the desired swing trajectories of the prosthetic can be calculated accordingly using the PHZD reconstruction method discussed in Sec. II.

IV. EXPERIMENTAL REALIZATION

Utilizing the optimal multi-contact gait generated in Sec. II and the control architecture introduced in Sec. III, we now have the framework to realize the main contribution of this paper experimentally on a custom-built prosthesis AMPRO to achieve dynamic multi-contact walking. The resulting walking using the real-time optimization-based controller will be compared with other controllers.

A. AMPRO and IMU sensing

AMPRO (AMBER Prosthetic) is a custom designed self-contained transfemoral prosthetic device, which includes two brushless DC motors to actuate both the ankle and knee contained transfemoral prosthetic device, which includes two

brushless DC motors to actuate both the ankle and knee

walking using the real-time optimization-based controller to achieve dynamic multi-contact walking. The resulting

paper experimentally on a custom-built prosthesis AMPRO

have the framework to realize the main contribution of this

B. Experiment Results

A PD controller $\mu_{pd}$ is first implemented to track the designed trajectories to achieve stable walking. Walking trials were performed on a treadmill providing a constant speed of 1.3 mph. The impedance parameters are estimated based on the experimental walking data obtained using the PD controller. The detailed estimation method can be referred to [31]. We then apply impedance control $\mu_{imp}$ as the feed-forward term while using the MIQP control $\mu_{qp}$ as the feedback term to track the desired joint trajectories. The resulting joint trajectories are shown in Fig. 6, and the experimental gait tiles along with the simulated prosthetic walking are shown in Fig. 7. A video of the resulting multi-contact walking can be seen at [2]. Therefore, utilizing the systematic methodology including gait design and optimization-based control, AMPRO successfully achieved stable and human-like multi-contact walking.

In particular, with the goal of showing the torque optimality of the proposed controller, different torque bounds (high torque 100 Nm for MIQPH and low torque 40 Nm MIQPL) for both $\mu_{MAX}$ and $\mu_{MAX}$ are tested during the experiment. We also compare it with an augmented control strategy, PD+Impedance (i.e., $\mu = \mu_{pd} + \mu_{imp}$), which also includes impedance control as a feed-forward term. The comparing results (including rms errors and power consumption) of using different controllers are shown in Fig. 8. From this figure, we can see that the tracking performances of both the ankle and knee are the best with MIQPH+Imp control. Importantly, the better performance doesn’t require more energy when compared with PD+Impedance control. Similar results can also be found when comparing MIQPL+Impedance and PD control. To summarize, we can conclude that the MIQP+Impedance controller has the best balanced performance between tracking and power requirements.

V. CONCLUSIONS

By leveraging a systematic methodology—including hybrid system models and real-time optimization-based controllers—this paper successfully translated the multi-contact behavior that is intrinsic in human locomotion from bipedal walking on AMBER2 to prosthetic walking on the prosthesis AMPRO. The performance of multiple controllers—utilized to track the generated optimal multi-contact gait—are compared with the real-time optimization
based controller resulting in the best overall performance. The obtained prosthetic walking is shown to capture the essentials of human walking both kinematically and kinetically, resulting in a smoother and more comfortable user experience when compared to flat-footed walking.

REFERENCES